STATISTICS – THE MEAN AND THE MEDIAN

❑ *WHOLE NUMBERS*

The Mean

Monica and Jarod are arguing over who is doing better in their Algebra class. Monica boasts the highest test score and claims she is doing better. But Jarod says his scores are more consistent, so he's doing better. How can we help them figure out who is doing better?

One way (of many) to settle this dispute is to calculate the ***mean*** of each person's test scores, and then simply compare the means to see who's done better. The mean of a set of numbers is just a fancy word for what most people call the *average*. The ***mean*** is calculated by adding all the numbers and then dividing that sum by how many numbers were added.

For example, assume Monica's test scores in the class were 78, 92, 99, 63, and 93. To find the **mean** test score, we divide the sum of the five tests by 5:

$$\text{Monica's mean} = \frac{78+92+99+63+93}{5} = \frac{425}{5} = \mathbf{85}$$

For Jarod's scores of 94, 83, 88, 91, and 89, we get his mean:

$$\text{Jarod's mean} = \frac{94+83+88+91+89}{5} = \frac{445}{5} = \mathbf{89}$$

So instead of arguing about all 10 test scores, and trying to decide if Monica's score of 99 makes her scores better than Jarod's, or trying to

estimate the damage done by Monica's score of 63, or arguing about whose grades are more consistent, we simply compute each student's mean score, and let the math decide: Jarod wins.

In business and science, the mean is often not a fair summary of data, so we now study another measure of center, the *median*.

The Median

The highway median is the "center divider," the portion of the highway that is in the middle of the lanes. Assuming a normal 6-lane highway, there are three lanes on each side of the median. In the same way, the ***median*** of a set of numbers is the middle number in the list of numbers.

There are two critical issues here. The first is that in order for the median to make any sense at all and truly be the middle number, we must first put the numbers in order. It doesn't matter whether we sort them in ascending (small to big) or descending (big to small) order. The second issue will be covered in homework problem 14.

For instance, let's calculate the **median** of Monica's test scores in the previous example. Notice that if we take her five scores in the order presented in the problem, the middle score would be 99:

78 92 99 63 93 the **wrong** way ☹

But that's absurd — who could possibly claim that 99 (the biggest score!) is the middle score, just because it's in the middle of the list? So let's not forget to put the five numbers in order first; for no particular reason (other than tradition), we'll sort them in ascending order:

63 78 92 93 99 the **right** way ☺

Now which is the middle score? Do you see that the 92 has two scores below it and two scores above it? 92 is the middle score, and therefore, **the median is 92**.

To help you keep track of the meanings of *mean* and *median*, notice that the words "me**d**ian" and "mi**d**dle" each have "d" as their third letter.

Contrasting the Mean and the Median

Consider the following set of numbers:

18 47 89 103 143

Add the five numbers together and divide by 5 to get a mean of 80. Also, since the numbers are already in ascending order, the median is 89.

Now watch what happens when we change the 143 at the end of the data listed above to 8,573 and leave the rest of the numbers alone:

18 47 89 103 **8,573**

Calculate the new mean and you should get 1,766, way bigger than the original mean of 80. But now check this out: The median is still 89! So, even though the mean jumped considerably when we increased the last number, the median stayed exactly the same.

Conclusion: Extremely small or extremely large numbers may wreak havoc on the mean, but may not affect the median.

Homework

1. Find the median of 37, 77, 58, 41, 67
2. Find the median of 53, 29, 3, 55, 98
3. Find the mean of 74, 68, 12, 54
4. Find the mean of 5, 27
5. Find the mean of 16, 41, 40, 70, 78
6. Find the median of 44, 30, 99

If you take Statistics, you'll find two notations for the ***mean***. The *population mean* is denoted by μ, the Greek letter mu (pronounced myoo). The *sample mean* is written $\bar{x}$ (read as *x*-bar).

7. Find the median of 63, 72, 99

8. Find the mean of 17, 10, 32, 65

9. Find the median of 29, 75, 65

10. Find the mean of 52, 32, 93

11. Find the mean of 8, 18

12. Find the median of 48, 73, 71, 2, 9, 15, 99

13. Your friend is calculating the mean of 50 numbers, where the smallest number in the bunch is 17 and the largest is 99.

    a. What would you say if your friend calculated a mean of 123?
    b. What about a mean of 88?
    c. What about a mean of 15?

14. You may have noticed something about the concept of median and all the homework problems on median: Every problem contained an odd quantity of numbers, namely either 3, 5, or 7 numbers. What if there's an even number of numbers? For example, how do we find the median of the four numbers (notice that they're already in ascending order)

    23 30 38 97

    where there's no middle number? We'll calculate the number halfway between the two numbers closest to the middle. In other words, we'll find the *mean* of 30 and 38, which is

    $$\frac{30+38}{2} = \frac{68}{2} = 34$$

    Thus, our conclusion is that **the median is 34**, even though 34 was not one of the original numbers. This is alright because we remember that the main feature of the median is that there are as many numbers below it as there are above it. And sure enough, two numbers (23 & 30) are below the median 34 and two numbers (38 & 97) are above it.

    And don't forget to put the numbers in order before you calculate the median.

15. Calculate the median of each set of numbers:

a. 27 25 100 18

b. 6 22 21 37 13 2

c. 50 15 12 9 12 8

d. 17 45 100 0

e. 34 113 287 77 89 50

f. 6 8 4 7 7 3 4 2

❑ *FRACTIONS*

Review of Mean and Median

Assume we are given a set of numbers. The **mean** is found by dividing the sum of the numbers by the number of numbers.

The **median** is found by listing the numbers in order (ascending or descending), and then selecting the one in the middle. If there is an even number of numbers, the median is found by calculating the *mean* of the two numbers closest to the middle.

EXAMPLE 1: **Find the mean:** $\frac{3}{4}$ $\frac{1}{2}$ $\frac{5}{6}$

Solution: We add the three numbers and divide by 3:

Step 1:
$$\begin{aligned} \tfrac{3}{4} &= \tfrac{9}{12} \\ \tfrac{1}{2} &= \tfrac{6}{12} \\ +\ \tfrac{5}{6} &= \tfrac{10}{12} \\ \hline & \tfrac{25}{12} \end{aligned}$$

Step 2:
$$\begin{aligned} & \tfrac{25}{12} \div 3 \\ = & \tfrac{25}{12} \div \tfrac{3}{1} \\ = & \tfrac{25}{12} \times \tfrac{1}{3} \\ = & \boxed{\tfrac{25}{36}} \end{aligned}$$

Question: How wise would it have been had we converted the sum $\frac{25}{12}$ into the mixed number $2\frac{1}{12}$ before dividing by 3?

EXAMPLE 2: **Find the median:** $\frac{3}{4}$ $\frac{1}{2}$ $\frac{5}{6}$

Solution: We line the numbers up in ascending order, from smallest to largest (largest to smallest would work, too), and then select the number in the **middle**.

To discern the relative sizes of the fractions, convert them to equivalent fractions with a common denominator and then line them up:

$\frac{3}{4}$ $\frac{1}{2}$ $\frac{5}{6}$ (the given data)

$\Rightarrow$ $\frac{9}{12}$ $\frac{6}{12}$ $\frac{10}{12}$ (common denominator, with new numerators)

$\Rightarrow$ $\frac{6}{12}$ $\frac{9}{12}$ $\frac{10}{12}$ (sort in ascending order)

The middle number is $\frac{9}{12}$, and so the median is

$\boxed{\frac{3}{4}}$

Homework

16. Find the **mean**:

a. $\frac{2}{3}$ $\frac{1}{4}$ $\frac{1}{2}$

b. $\frac{9}{10}$ $\frac{5}{12}$

c. $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$

d. $\frac{1}{9}$ $\frac{1}{8}$ e. $\frac{5}{6}$ $\frac{2}{3}$ $\frac{7}{12}$ f. $2\frac{1}{2}$ $\frac{3}{4}$ $1\frac{5}{8}$

g. $3\frac{1}{4}$ $2\frac{2}{3}$ h. $1\frac{1}{2}$ $\frac{4}{5}$ $\frac{5}{6}$ i. $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{6}$

17. Find the **median**:

a. $\frac{2}{3}$ $\frac{1}{4}$ $\frac{1}{2}$ b. $\frac{5}{6}$ $\frac{2}{3}$ $\frac{7}{12}$ c. $2\frac{1}{2}$ $\frac{3}{4}$ $1\frac{5}{8}$

d. $\frac{1}{7}$ $\frac{1}{5}$ $\frac{1}{9}$ e. $\frac{2}{3}$ $\frac{4}{5}$ $\frac{1}{2}$ f. $\frac{5}{8}$ $\frac{1}{2}$ $\frac{3}{4}$

g. $\frac{1}{5}$ $\frac{3}{10}$ $\frac{2}{11}$ h. $\frac{4}{5}$ $\frac{3}{4}$ $\frac{1}{2}$ i. $\frac{7}{15}$ $\frac{11}{30}$ $\frac{2}{5}$

❑ *DECIMALS*

Putting Decimals in Order

As you know, the key to finding the median of a set of numbers is the ability to put those numbers in numerical order. The method most students find easiest to understand is converting the decimals to money.

For example, suppose we need to list the following numbers in ascending order (smallest to biggest):

2.3 0.129 0.07 0.8 5

Translate each number into currency format:

\$2.30 \$0.129 \$0.07 \$0.80 \$5.00

[Note that the second number, \$0.129, is a tad short of \$0.13.] Now we put these money amounts in order from smallest to biggest:

\$0.07 \$0.129 \$0.80 \$2.30 \$5.00

Since \$0.80 is now the middle number, we conclude that the median of the original set of numbers is **0.8**.

Homework

18. Find the **mean** (NO rounding):

a. 2.3 8.9

b. 1.1 0.5 2.3

c. 0.02 0.2 2

d. 1.1 2.2 3.3 4.4

e. 0.23 0.44 1.1 0.092

f. 0.2 9.1 2.33 8 5.023

19. Find the **median**:

a. 0.1 0.11 0.01

b. 1.1 0.5 2.3

c. 0.02 0.2 2

d. 2.3 4.5 4.05 2.003 .999

e. 0.03 0.04 0.4 0.3 0.9

f. 0.01 0.002 1.0005 0.03 0.017

20. Find the **median**:

a. 2.3 0.02 1.77 0.001

b. 23.9 14.89 17.9 30

c. 0.07 0.8 0.006 2

d. 2.3 2 1.99 5

e. 3.4 5.6 1.89 3.77

f. 0.09 0.045 2 1.17

g. 23.5 12.99 3.567 0.07

h. 0.25 0.078 0.667 0.0099

i. 23.9 32.007 18.345

j. 0.23 0.32 0.4 0.003 0.09 0.1

Solutions

1. 58 **2**. 53 **3**. 52 **4**. 16 **5**. 49 **6**. 44

7. 72 **8**. 31 **9**. 65 **10**. 59 **11**. 13 **12**. 48

13. Only the 88 makes sense. Ask if you don't understand.

14. This was just an example, not a problem.

15. a. 26 b. 17 c. 12 d. 31 e. 83 f. 5

The new "zipper" **MEDIAN** on the Golden Gate Bridge

16. a. $\frac{17}{36}$ b. $\frac{79}{120}$ c. $\frac{77}{240}$ d. $\frac{17}{144}$ e. $\frac{25}{36}$ f. $1\frac{5}{8}$

g. $2\frac{23}{24}$ h. $1\frac{2}{45}$ i. $\frac{11}{16}$

17. a. $\frac{1}{2}$ b. $\frac{2}{3}$ c. $1\frac{5}{8}$ d. $\frac{1}{7}$ e. $\frac{2}{3}$ f. $\frac{5}{8}$

g. $\frac{1}{5}$ h. $\frac{3}{4}$ i. $\frac{2}{5}$

18. a. 5.6 b. 1.3 c. 0.74 d. 2.75 e. 0.4655 f. 4.9306

19. a. 0.1 b. 1.1 c. 0.2 d. 2.3 e. 0.3 f. 0.017

20. a. 0.895 b. 20.9 c. 0.435 d. 2.15 e. 3.585 f. 0.63
g. 8.2785 h. 0.164 i. 23.9 j. 0.165

"Life does not need to be changed.

Only your intent and actions do."

– *Swami Rama*